Written Exam for the M.Sc. in Economics Summer 2011
Fixed Income Derivatives
Final Exam - Master's Course

9-11th of April 2011

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.
If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.
The paper must be uploaded as <u>one PDF document</u> (including the standard cover and the appendices). The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Absalon.

Focus on Exam Cheating

In case of presumed exam cheating, which is observed by either the examination registration of the respective study programmes, the invigilation or the course lecturer, the Head of Studies will make a preliminary inquiry into the matter, requesting a statement from the course lecturer and possibly the invigilation, too. Furthermore, the Head of Studies will interview the student. If the Head of Studies finds that there are reasonable grounds to suspect exam cheating, the issue will be reported to the Rector. In the course of the study and during examinations, the student is expected to conform to the rules and regulations governing academic integrity. Academic dishonesty includes falsification, plagiarism, failure to disclose information, and any other kind of misrepresentation of the student's own performance and results or assisting another student herewith. For example failure to indicate sources in written assignments is regarded as failure to disclose information. Attempts to cheat at examinations are dealt with in the same manner as exam cheating which has been carried through. In case of exam cheating, the following sanctions may be imposed by the Rector:

1. A warning
2. Expulsion from the examination
3. Suspension from the University for at limited period or permanent expulsion.

The Faculty of Social Sciences
The Study and Examination Office
October 2006

Introduction

The exam questions are accompanied by a spreadsheet with data that can be downloaded from the course website. Also, you need to download the latest version of `fidAnalytics.xla`. Make sure to read all the questions as well as the data spreadsheet carefully. The purpose of the exam is to test your ability to price and risk manage derivatives. In order to receive the grade 12, you must demonstrate the ability to implement and use the models and methodologies covered in the course.

GOOD LUCK!

Formalities

Do NOT include your name or any other personal details. The deadline for uploading your solution is Monday, April 11th at 10.00. Late solutions will not be accepted.

If you believe that you have found any errors or unclarities in the assignment, the lecturer can be reached via e-mail at martin.linderstroem@gmail.com. You are not allowed to discuss the exam with anyone other than the lecturer.

Excel/VBA settings

In order to avoid any differences in results stemming from numerical issue alone, you should set your Excel `SOLVER` to have the following settings:

- Max Time=300
- Iterations=300
- Precision=0.000001
- Tolerance=5%
- Convergence=0.000001
- Automatic Scaling=Yes

1 The Swap Curve

1.1 Estimating the swap curve

Using the market data found in the spreadsheet, estimate the forward and discounting zero coupon curves for the EUR market using hermite interpolation. You should use the knot points listed in the Data sheet. The listed CCS par spreads are quotes against 3M USD LIBOR flat. You should estimate the curves under the assumption that you are representing a bank that can fund itself at 3M USD LIBOR.

Report your estimated knot point rates with 4 digits (e.g. 4.0000%). Plot the continuously compounded Act/365 zero coupon (forward and discounting) rates as well all 3M forward xIBOR rates (Act/360, MF) in 3M spacings out to January 1st, 2041. Also, plot the forward 3M CCS breaks (Act/360, MF) in 3M spacings out to January 1st, 2041. The zero- and forward xIBOR rates should be shown on the same axis in percent, while the forwards CCS breaks should be shown its own axis in basis points.

1.2 Marking swap portfolios to market

In the data sheet, you will find the details for two portfolios of IRSs. What is the mid-market PV of these two portfolios on the curves calibrated above? Report the PV per individual swap as well as the portfolio total in two tables.

1.3 Reporting risk on swap portfolios

For each of the swap portfolios, report both the (forward and discount curve) zero rate delta vector as well as the market rate delta vectors. Looking at the market rate delta vector, how would you verbally characterize the risk for of each of the two portfolios (i.e. are you positioned for higher or lower rates, steeper or flatter curves, higher or lower cross currency basis spreads)?

1.4 Quantifying portfolio risk

Rather than just relying on the verbal formulation from above on your risk profile, you should now try to quantify it. Specifically, you should use the time series data found in the data sheet to estimate 95% 1-day Value-at-Risk (VaR) and ditto Expected Shortfall (ES). Report the two risk figures for each of the portfolios — which portfolio would you judge to be the riskiest?

Hint: You should combine the market rate delta vectors found above with the time series of 1-day changes in swap rates and basis spreads to estimate a profit/loss distribution from historical data. Use this estimated distribution to estimate VaR and ES.

2 Pricing CMS products

2.1 Calibrating SABR models

Using the given (forward and discounting) zero rate curves found in the data sheet, you should calibrate a total of 3 SABR models. Use the listed swap conventions when calculating the relevant forward swap rates. You should fix $\alpha = 52\%$ and thus effectively

only use α, ϵ and ρ to calibrate the models. Report your calibrated SABR parameters in percent with two decimals (e.g. 35.00%).

Hint: Remember to use Act/365 to calculate the time-to-expiration for each swaption expiry.

2.2 Pricing a CMS swap

Use the SABR calibrated calibrated above to price a 5Y CMS swap written on the 30Y index. You should interpolate linearly between the calibrated SABR models to find the relevant set of parameters for each fixing date.

The CMS swap is the plain vanilla variant, where a series of 3M EURIBOR fixings are swapped against 30Y CMS fixings. The CMS swap has 2B spot start and a maturity of 5Y, payments are made in-arrears every 3M (adjusted MF) relative to Act/360.

What is the spread to be paid on the 3M EURIBOR leg for the swap to be valued at par? Report your finding as a number of basis points with one decimal.

Hint: Price the 3M floating rate payment stream and the CMS payment stream independently to find a net value and spread this on the 3M EURIBOR leg. Use the `fidGenerateSchedule` function to create the payment schedule for the CMS swap.

2.3 Static CMS replication

Use the existing `fidCmsSwapletPv` function as a starting point to build a new and expanded version called `fidCmsSwapletPvNew`. The new function should be expanded such that the upper bound for the integration (`IntUpBound`) can be supplied as an input to the function. Use this new function to repeat the pricing from above for `IntUpBound`= $\{5\%, 10\%, 15\%, 20\%, 30\%\}$.

Report the par CMS spreads corresponding to each upper bound in a table. Comment on your findings, how and why does the CMS pricing depend on this input?

Hint: Basically, you are asked to repeat the CMS pricing a total of 5 times corresponding to each choice of integration upper bound. The easiest way to calculate this is to use the `TABLE` function in Excel. Note that this is a slow calculation, so you should set your sheet in calculation manual mode and have some patience with your PC.

3 The capped floater

For a number of years, a popular product in the Danish mortgage market was capped floater bonds. In such a bond, the borrowers (home owners) pays coupons to the bondholders of the following type: Coupon= min(Borrower strike,6M CIBOR+Spread). For simplicity, we will look at the interest-only version (i.e. the version of the bond where there are no redemptions of the notional over the life of the bond). In this case, we can think the capped floater bond as a long position in a zero coupon bond (repaying the notional), a long position in a floating rate payment stream (plus a spread) on a bullet notional and a short position in a cap on a bullet notional.

The specific bond we will look at has 10 year maturity and a borrower spread over 6M CIBOR of 60 bps. Finally, the borrower pays a maximum coupon of 5%. The bond pays coupons semi annually (rolled MF) according to Act/360 (the borrower spread and strike are quoted on the same basis). As we are standing on a coupon payment date, there is no accrued interest (clean and dirty prices are identical). For simplicity, we will be assuming

that all the caplets in the embedded cap can be priced on "same" SABR model, i.e. you should use the listed σ_0, α, ϵ and ρ for for all caplets but naturally use different time-to-expiration for each individual SABR model corresponding to the individual forward rates. Again, you should use Act/365 to calculate the time-to-expiration for each SABR model.

3.1 The cap in the capped floater bond

What plain vanilla cap strike does the implicit cap above correspond to? Using the given zero curves and the given SABR model, what is the mid-market value of the cap on a notional of DKK 100m? You should set up the cap such that is has 2B spot start and a maturity of 10Y.

3.2 The risk profile of the capped floater

Calculate the net (forward plus discounting) zero rate delta vector for a long position in the capped floater bond of DKK 100m. Plot you findings in column chart.

Hint: You should work our the value of each component of the bond (as they are priced on the swap curve with the given SABR model) and use this to compute the risk profile. As you are using a SABR model to value the cap, remember to let your caplet volatilities depend on the curve shifts.

3.3 Capped floater asset swap spread

With a clean price of 97.50, what is the par-par asset swap spread against 6M CIBOR for the capped floater bond? What does this asset swap spread intuitively tell us about the bond?

3.4 Capped floater product development

Suppose now, that the same mortgage institute who have issued the bond above want to expand their product range. They want to offer another capped floater variant to homeowners. In particular, they want to offer a variant where the borrower strike (i.e. the max coupon that the borrower can pay) is instead 6%.

What should the borrower spread be in order to result in the same par-par asset swap spread, if the mortgage institute want to issue the bond at a price of 99.50? Comment on the size of the borrower spread relative to the 5% variant.

Hint: Besides the change in borrower strike and -spread, the two capped floater bonds are exactly identical. You should simply vary the borrower spread and (and thus cap strike) until you match the asset swap spread.

3.5 A callable capped floater

For the capped floater bonds issued in the Danish mortgage bond market it has customary to embed yet another option — the 105 call option. Specifically, this option gives the homeowners the right (but not the obligation) to buy back the bond (i.e. buy back their loan) at a price of 105. Intuitively, under which scenarios does this 105 call option have value?

Hint: You are not asked to price the 105 option (as it requires full term structure modelling), rather you should think about what general rate-, asset swap spread- and volatility scenarios would cause the price of the bond to rise to 105.